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Euclidean Thermal Green Functions of Photons in Generalized Euclidean Rindler Spaces for any Feynman-like Gauge.

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Abstract:

The thermal Euclidean Green functions for Photons propagating in the Rindler wedge are computed employing an Euclidean approach within any covariant Feynman-like gauge. This is done by generalizing a formula which holds in the Minkowskian case. The coincidence of the found ($\beta = 2\pi$)-Green functions and the corresponding Minkowskian vacuum Green functions is discussed in relation to the remaining static gauge ambiguity already found in previous works. Further generalizations to more complicated manifolds are discussed. Ward identities are verified in the general case.

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1   Introduction

In principle, from the thermal Euclidean Rindler Green functions of quantum fields (also with spin) and employing standard different imaginary time analytic continuations, one can get either the *thermal* Feynman propagators in the Rindler space (for example see [1] for the scalar case and [2] for the vectorial case) or the *non-thermal* Feynman propagators around a cosmic string at least for $\beta \leq 2\pi$ (for example see [3] where generalizations to the case $\beta > 2\pi$ are also studied). In fact, the (time periodic) Euclidean Rindler metric reads

$$ds^2 = g_{ab}dx^a dx^b = \rho^2 d\theta^2 + d\rho^2 + dy^2 + dz^2 \tag{1}$$

where $\rho \geq 0$ and $0 \leq \theta \leq \beta \equiv 0$. We may interpret θ as the imaginary Euclidean Rindler time and thus we may come back in the Lorentzian Rindler space performing an analytic continuation such as $\theta \rightarrow -i\tau$. As it is well-known, the (Lorentzian) Rindler metric is the metric seen by uniformly accelerated observers, corresponding to the trajectories with $\rho, y, z = $ constants, in the portion of Minkowski space called (right) Rindler wedge. The relation between Minkowski coordinates t, x, y, z and Rindler coordinates τ, ρ, y, z reads $t = \rho \sinh \tau$, $x = \rho \cosh \tau$, $\rho > 0$, $\tau \in I\!R$. The right Rindler wedge is then defined as $x > |t|$.

The value of β defines the temperature $T = 1/\beta$ of the field quantum state for such observers. Indifferently, we can perform the continuation $z \rightarrow -it$ obtaining the metric in the Lorentzian section of a cosmic string spacetime with time t. The constant β is now connected to the linear mass density of the string $\mu = (2\pi - \beta)/8\pi G$.

For the value $\beta_U = 2\pi$, corresponding to the well-known Unruh temperature [4] as well as the disappearance of the conical singularity at $\rho = 0$ from the Euclidean and the Lorentzian string manifold, Minkowski vacuum Green functions and ($\beta = 2\pi$)-Rindler Green functions are expected to coincide. This is due to the (local) coincidence of the corresponding quantum states. A vast literature exists on this topic (see [4, 5] and [6] for a recent review) also related to the physics of accelerated detectors [7], [8]. In other contexts, the metric (1) is also interesting because is a well-tried approximation of the (Euclidean-section) metric near a Schwarzschild black hole event horizon. The particular value β_U is then related to Hawking's temperature of the black hole [7].

Let us focus our attention on the Euclidean Green function for a photon field propagating in a Rindler wedge which has the correct periodic behaviour due to the Lorentzian KMS condition [9, 1]. By the canonical quantization, summing over images and thus continuing into the imaginary time, Moretti and Vanzo obtained in [11, 2] the following (thermal) photon *Schwinger function*

in the *Feynman gauge*, α being defined by $2\rho\rho' \cosh\alpha = \rho^2 + \rho'^2 + |\mathbf{x} - \mathbf{x}'|^2$:

$$S_{\beta\,\theta\theta'}(x, x') = \frac{1}{4\pi\beta\sinh\alpha} \frac{\cos\left(\frac{2\pi}{\beta}(\theta - \theta')\right)\sinh\alpha + \sinh\left[\left(\frac{2\pi}{\beta} - 1\right)\alpha\right]}{\cosh\left(\frac{2\pi}{\beta}\alpha\right) - \cos\left(\frac{2\pi}{\beta}(\theta - \theta')\right)} + \frac{1}{4\pi\beta'}, \tag{2}$$

$$S_{\beta\,\rho\rho'}(x, x') = \frac{1}{\rho\rho'}S_{\beta\,\theta\theta'}(x, x'), \tag{3}$$

$$S_{\beta\,\theta\rho'}(x, x') = \frac{-1}{4\pi\beta\rho'} \frac{\sin\left(\frac{2\pi}{\beta}(\theta - \theta')\right)}{\cosh\left(\frac{2\pi}{\beta}\alpha\right) - \cos\left(\frac{2\pi}{\beta}(\theta - \theta')\right)}, \tag{4}$$

$$S_{\beta\,\rho\theta'}(x, x') = -\frac{\rho'}{\rho}S_{\beta\,\theta\rho'}(x, x'), \tag{5}$$

$$S_{\beta\,yy'}(x, x') = S_{\beta\,zz'}(x, x') = S_\beta(x, x'). \tag{6}$$

where $\beta' = \beta$ (see discussion below). In [10] similar results have been obtained in the Lorentzian background of a cosmic string. An equivalent expression for the case $\beta = \beta'$ is

$$S_{\beta\,aa'}(x, x') = V_{\beta\,aa'}(x, x')\ S_\beta(x, x'), \tag{7}$$

where

$$V_{\beta\,\theta\theta'} = \rho\rho'V_{\beta\,\rho\rho'} = \rho\rho'\cosh\alpha,$$
$$V_{\beta\,\theta\rho'} = -\rho\rho'^{-1}V_{\beta\,\rho\theta'} = -\rho\sin(\frac{2\pi}{\beta}(\theta - \theta'))\frac{\sinh\alpha}{\sinh\frac{2\pi\alpha}{\beta}}$$
$$V_{\beta\,yy'} = V_{\beta\,zz'} = 1,$$

all the remaining bi-vectors vanish. Here and above the function:

$$S_\beta(x, x') = \frac{1}{4\pi\beta\rho\rho'\sinh\alpha}\left[\frac{\sinh\left(\frac{2\pi}{\beta}\alpha\right)}{\cosh\left(\frac{2\pi}{\beta}\alpha\right) - \cos\left(\frac{2\pi}{\beta}(\theta - \theta')\right)}\right] \tag{8}$$

is a well-known Rindler thermal Schwinger function for a massless scalar field obtained by rotating into the imaginary time the Feynman propagator. This propagator is obtained by the sum over images after the canonical quantization. These functions are Euclidean *Green functions* in the sense that they satisfy the respective Green identities:

$$\nabla^a\nabla_a S_\beta(x, x') = -g(x)^{-1/2}\,\delta(x, x'), \tag{9}$$

$$\nabla^a\nabla_a S_{\beta\,bb'}(x, x') = -g_{bb'}(x)\,g(x)^{-1/2}\,\delta(x, x'). \tag{10}$$

In the limit β (and β') $\to +\infty$ one recovers the non-thermal Schwinger functions calculated by the canonical quantization [2]. The parameter β' which appears in $S_{\beta\,\theta\theta'}$ and $S_{\beta\,\rho\rho'}$ can be chosen different from the time period β with no effect for every physical quantity. This parameter gives raise to an ambiguity in defining all the Green functions[2]. As discussed in

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[2]This is not a simple ambiguity due to an unfixed added constant to the Green functions because the term containing β' is a function of ρ and ρ' in $S_{\beta\,\rho\rho'}$.

[11, 2] this is just a remaining static gauge ambiguity. Moreover, due to this gauge ambiguity we have not the expected coincidence of Minkowskian and ($\beta = 2\pi$)-Rindler Green function *also* for $\beta' = \beta(= 2\pi)$. One must take $\beta' = +\infty$ in order to reach such a coincidence. However, the physics seems to be safe because this coincidence is restored, non depending on the value of β', for the Lorentzian two-point function as well as the Feynman propagator when they work on Lorentz and physical photon wavefunctions [11, 2]. The presence of several problems in studying photon theory in a Rindler wedge (also related with the gauge invariance) has been encountered in other contexts [12, 13]. As far as the Green functions are concerned, it is interesting to evaluate the photon Green functions employing methods different from the sum over images and check if a different value for β' is selected (for instance the more natural value $\beta' = +\infty$). Furthermore, it is also intersting to check whether or not the previous problems are specific features of the Feynman gauge only.

The result of the present paper, which generalizes a well-known formula holding in the Minkowskian case, is a development of the previous papers because the Green function expression we shall get holds for any (Feynman-like) covariant gauge and for a generalized Euclidean Rindler space $\mathcal{M} \times I\!\!R^2$ endowed with the natural product metric. $\mathcal{M}$ is a generally curved two-dimensional Euclidean manifold which, in the Rindler case, reduces to $\mathcal{C}_\beta$, i.e. a cone of an angular deficit $2\pi - \beta$. However, differently from [11] and [2], our result will arise from a direct Euclidean approach instead of the canonical quantization. For this reason we prefer the term Euclidean Green function instead of Schwinger function.

We shall see that the Green functions we found in the Rindler wedge contain the same static gauge ambiguity previously discussed only for the Feynman gauge and we shall get the coincidence of ($\beta = 2\pi$)-Euclidean Rindler Green functions and Euclidean Minkowski Green functions except for the usual discrepancy corresponding to the presence of unphysical photons in the Lorentzian theory. Finally, we shall prove the Ward identities for the general case.

2   Euclidean Approach for Feynman Gauge

Let us start by considering an Euclidean Rindler wedge of time period β: $\mathcal{C}_\beta \times I\!\!R^2$. From now on Latin indices as a, b, c, d are for the whole manifold, Greek indices are for the pure cone and Latin indices as $i, j... = y, z$ are for the remaining $I\!\!R^2$. The Euclidean Maxwell equations, in a generic Feynman-like covariant gauge parametrized by $\eta \in I\!\!R$, read

$$\left[\delta_a^b \Delta + R_a^b - (1 - \frac{1}{\eta})\nabla_a \nabla^b\right] A_b = 0 \,. \tag{11}$$

$\Delta = \nabla_c \nabla^c$ is the Laplace-Beltrami vectorial operator built up using covariant derivatives. Obviously, when $\eta = 1$ one recovers the usual Feynman gauge Maxwell equations.

In the case of the Rindler space $R_a^b = 0$ holds except for $\rho = 0$, however we shall employ normal modes which vanish in those points and thus we shall omit R_a^b. Later, we shall consider also that term in more complicated manifolds employing the Hodge-de Rham formalism.

We want to get a Green function of the previous operator by the usual sum of normal modes products. Let us first consider the case $\eta = 1$ and prove that this leads us to the Schwinger function in Eq.(7). An useful complete eigenfunctions set of the vectorial Laplace-Beltrami operator (corresponding to $\eta = 1$ and $R_a^b = 0$ in Eq.(11)) is that discussed by Kabat [12]:

$$A_a^{(I,n\lambda\mathbf{k})} \ := \ (0, 0, \phi, 0), \tag{12}$$

$$A_a^{(II,n\lambda\mathbf{k})} \ := \ (0, 0, 0, \phi), \tag{13}$$

$$A_a^{(III,n\lambda\mathbf{k})} \ := \ \frac{\sqrt{g}}{\lambda}\epsilon_{\mu\nu}\partial^\nu\phi = \frac{1}{\lambda}(\rho\partial_\rho\phi, -\frac{1}{\rho}\partial_\theta\phi, 0, 0), \tag{14}$$

$$A_a^{(IV,n\lambda\mathbf{k})} \ := \ \frac{1}{\lambda}\partial_\mu\phi = \frac{1}{\lambda}(\partial_\theta\phi, \partial_\rho\phi, 0, 0) \tag{15}$$

$\sqrt{g}\epsilon_{\mu\nu}$ is the Levi-Civita pseudo-tensor on the cone and $\phi = \phi^{(n\lambda\mathbf{k})}(x)$ defines the complete scalar eigenfunctions set of a self-adjoint extension of the scalar Laplacian on $C_\beta \times I\!\!R^2$ with eigenvalues $-(\lambda^2 + \mathbf{k}^2)$ [14, 12]:

$$\phi^{(n\lambda\mathbf{k})}(x) \ := \ \frac{1}{2\pi}\sqrt{\frac{\lambda}{\beta}}e^{i\mathbf{k}\mathbf{x}}e^{i\frac{2\pi n}{\beta}\theta}J_{\nu_n}(\lambda\rho), \quad n \in Z\!\!\!Z ; \ \lambda \in I\!\!R^+; \ \mathbf{k} = (k_y, k_z) \in I\!\!R^2 . \tag{16}$$

Here J_{ν_n} is the Bessel function of first kind and $\nu_n = \frac{2\pi|n|}{\beta}$. The previous modes are normalized according to

$$\int d^4x \sqrt{g}\, g^{ab} A_a^{(m',n'\lambda'\mathbf{k}')*} A_b^{(m,n\lambda\mathbf{k})} \ = \ \delta_{m'm}\delta_{n'n}\delta^{(2)}(\mathbf{k} - \mathbf{k}')\delta(\lambda - \lambda') , \tag{17}$$

$$\int d^4x \sqrt{g}\phi^{(n'\lambda'\mathbf{k}')*}\phi^{(n\lambda\mathbf{k})} \ = \ \delta_{n'n}\delta^{(2)}(\mathbf{k} - \mathbf{k}')\delta(\lambda - \lambda') , \tag{18}$$

In the considered case $\eta = 1$, all the previous eigenfunctions of the vectorial Laplacian correspond to the eigenvalues $-(\lambda^2 + \mathbf{k}^2)$. A Green function of this vectorial operator, hence satisfying Eq.(10), can be built up by the found modes as:

$$G_{\beta\,aa'}(x, x') \ := \ \int_0^{+\infty} d\lambda \int_{I\!\!R^2} d\mathbf{k} \sum_{n \in Z\!\!\!Z} \sum_{m=I}^{IV} \frac{A_a^{(m,n\lambda\mathbf{k})}(x)A_a^{(m,n\lambda\mathbf{k})*}(x')}{\lambda^2 + \mathbf{k}^2} ; \tag{19}$$

we can equivalently write

$$G_{\beta\,aa'}(x, x') \ = \ \int_0^{+\infty} \frac{d\lambda}{\lambda^2} \int_{I\!\!R^2} d\mathbf{k} \sum_{n \in Z\!\!\!Z} D_{aa'} \frac{\phi^{(n\lambda\mathbf{k})}(x)\phi^{(n\lambda\mathbf{k})*}(x')}{\lambda^2 + \mathbf{k}^2} , \tag{20}$$

where we used the definitions:

$$D_{\theta\theta'} = \rho\rho' D_{\rho\rho'} = \partial_\theta \partial_{\theta'} + \rho\rho' \partial_\rho \partial_{\rho'} ;$$

$$D_{\theta\rho'} = -\rho\rho'^{-1} D_{\rho\theta'} = \rho'^{-1} [\partial_\theta \rho' \partial_{\rho'} - \partial_{\theta'} \rho \partial_\rho] ;$$

$$D_{yy'} = D_{zz'} = \lambda^2 .$$

All the remaining components vanish. By employing the following regularization:

$$\lambda^{-2}(\lambda^2 + \mathbf{k}^2)^{-2} = \lambda^{-2}(\mathbf{k}^2 + \epsilon^2)^{-2} - (\mathbf{k}^2 + \epsilon^2)^{-2}(\lambda^2 + \mathbf{k}^2)^{-2} \quad \text{as} \quad \epsilon \to 0 ,$$

one can obtain as far as the conical components are concerned:

$$G_{\beta\,\mu\mu'}(x, x') = -\int_0^{+\infty} d\lambda \int_{I\!\!R^2} d\mathbf{k} \sum_{n\in\mathbb{Z}} \frac{D_{\mu\mu'}}{\mathbf{k}^2 + \epsilon^2} \frac{\phi^{(n\lambda\mathbf{k})}(x)\phi^{(n\lambda\mathbf{k})*}(x')}{\lambda^2 + \mathbf{k}^2} +$$
$$+ \int_0^{+\infty} d\lambda \int_{I\!\!R^2} d\mathbf{k} \sum_{n\in\mathbb{Z}} \frac{D_{\mu\mu'}}{\mathbf{k}^2 + \epsilon^2} \frac{\phi^{(n\lambda\mathbf{k})}(x)\phi^{(n\lambda\mathbf{k})*}(x')}{\lambda^2} . \tag{21}$$

The second term in the right hand side of Eq.(21) can be written:

$$\left(\frac{1}{4\pi^2} \int_{I\!\!R^2} d\mathbf{k} \frac{e^{\mathbf{k}(\mathbf{x}-\mathbf{x}')}}{\mathbf{k}^2 + \epsilon^2} \right) D_{\mu\mu'} \int_0^{+\infty} d\lambda \sum_{n\in\mathbb{Z}} \frac{\varphi^{(n\lambda)}(\theta, \rho)\varphi^{(n\lambda)*}(\theta'\rho')}{\lambda^2} , \tag{22}$$

where

$$\varphi^{(n\lambda)}(\theta, \rho) = \sqrt{\frac{\lambda}{\beta}} e^{i\frac{2\pi n}{\beta}\theta} J_{\nu_n}(\lambda\rho) \tag{23}$$

is an eigenfunction of the scalar Laplacian on the pure cone $\mathcal{C}_\beta$ with eigenvalue $-\lambda^2$. Hence, the remainder after $D_{\mu\mu'}$ in Eq.(22) coincides with a Green function of the Laplacian on the pure cone:

$$G_\beta^c(x, x') = -(2\beta)^{-1} \ln |\rho^2 + \rho'^2 - 2\rho\rho' \cos(\theta - \theta')| .$$

Furthermore, by a direct check on the right hand side of the previous expression, one finds:

$$D_{\mu\mu'} G_\beta^c(x, x') = 0 ,$$

and thus only the first term in the right hand side of Eq.(21) survives. Then we have:

$$G_{\beta\,\mu\mu'}(x, x') = -\int_0^{+\infty} d\lambda \int_{I\!\!R^2} d\mathbf{k} \sum_{n\in\mathbb{Z}} \frac{D_{\mu\mu'}}{\mathbf{k}^2 + \epsilon^2} \frac{\phi^{(n\lambda\mathbf{k})}(x)\phi^{(n\lambda\mathbf{k})*}(x')}{\lambda^2 + \mathbf{k}^2} . \tag{24}$$

As we expect from the general theory, one can prove the further identity

$$G_\beta(x, x') := \int_0^{+\infty} d\lambda \int_{I\!\!R^2} d\mathbf{k} \sum_{n\in\mathbb{Z}} \frac{\phi^{(n\lambda\mathbf{k})}(x)\phi^{(n\lambda\mathbf{k})*}(x')}{\lambda^2 + \mathbf{k}^2} \equiv S_\beta(x, x') , \tag{25}$$

where the latter right hand side is just the scalar Schwinger function defined in Eq.(8), obtained from the canonical quantization and the sum over images. This proves, through Eq.(20), that

$G_{\beta\,ij}(x, x') = S_{\beta\,ij}(x, x')$. We can employ Eq.(25) to further develop the expression in Eq.(24). Following the same way used in [2] for the non-thermal case we can write Eq.(24) as

$$G_{\beta\,\mu\mu'}(x, x') = -\frac{1}{2\pi} \int_{I\!\!R^2} d\mathbf{x}'' \ln \frac{|\mathbf{x}''|}{\mu_0} D_{\mu\mu'} S_\beta(\theta - \theta', \rho, \rho', \mathbf{x}'' - (\mathbf{x} - \mathbf{x}')) , \qquad (26)$$

where μ_0 is an unimportant constant defined in [2]. Moreover we can prove the following identities:

$$
\begin{aligned}
D_{\theta\theta'} S_\beta &= \rho\rho' D_{\rho\rho'} S_\beta = \rho\rho' \nabla_\mathbf{x}^2 [\cosh\alpha\, S_\beta(\theta, \theta', \rho, \rho', \mathbf{x})] , \\
D_{\theta\rho'} S_\beta &= -\rho'\rho^{-1} D_{\rho\theta'} S_\beta = -\rho \sin(\frac{2\pi}{\beta}(\theta - \theta')) \nabla_\mathbf{x}^2 [\sinh\alpha\, S_\beta(\theta, \theta', \rho, \rho', \mathbf{x})/\sinh\frac{2\pi\alpha}{\beta}] .
\end{aligned}
$$

Using these in Eq.(26) and reminding also that for smooth $I\!\!R^2$ functions ($c \in I\!\!R^+$ fixed):

$$\int_{I\!\!R^2} d\mathbf{x} \ln(|\mathbf{x}|/c) \nabla_\mathbf{x}^2 g(\mathbf{y} - \mathbf{x}) = -2\pi g(\mathbf{y}) ,$$

one can prove that

$$G_{\beta\,\mu\mu'}(x, x') = S_{\beta\,\mu\mu'}(x, x')$$

(where we supposed $\beta' = \beta$) by a direct comparison with the expression of $S_{\beta\,\mu\nu}(x, x')$ given in Eq.(7). Summarizing, we have finally proved that

$$G_{\beta\,aa'}(x, x') = S_{\beta\,aa'}(x, x') \qquad (27)$$

Thus, the same Green function calculated through the canonical quantization and the sum over images re-arises, with the same value β for β'. The non-thermal Schwinger function obtained by the canonical quantization arises as the limit as $\beta \to +\infty$ of $G_{\beta\,aa'}$.

3 Euclidean Approach for any Feynman-like Covariant Gauge

Let us now consider the case $\eta \neq 1$ in the operator in the left hand side of Eq.(11). An useful set of eigenfunctions normalized as in Eq.(17) and built up by linear combinations of the previous Kabat's modes reads [13]:

$$
\begin{aligned}
A_a^{(I,n\lambda\mathbf{k})} &= \frac{1}{k}\epsilon_{ij}\partial^j \phi = \frac{1}{k}(0, 0, ik_z \phi, -ik_y \phi), & (28) \\
A_a^{(II,n\lambda\mathbf{k})} &= \frac{\sqrt{g}}{\lambda}\epsilon_{\mu\nu}\partial^\nu \phi = \frac{1}{\lambda}(\rho\partial_\rho \phi, -\frac{1}{\rho}\partial_\theta \phi, 0, 0), & (29) \\
A_a^{(III,n\lambda\mathbf{k})} &= \frac{1}{\sqrt{\lambda^2 + \mathbf{k}^2}}(\frac{k}{\lambda}\partial_\mu - \frac{\lambda}{k}\partial_i)\phi = \frac{1}{\sqrt{\lambda^2 + \mathbf{k}^2}}(\frac{k}{\lambda}\partial_\theta \phi, \frac{k}{\lambda}\partial_\rho \phi, -\frac{\lambda}{k}\partial_y \phi, -\frac{\lambda}{k}\partial_z \phi), & (30) \\
A_a^{(IV,n\lambda\mathbf{k})} &= \frac{1}{\sqrt{\lambda^2 + \mathbf{k}^2}}\partial_a \phi = \frac{1}{\sqrt{\lambda^2 + \mathbf{k}^2}}(\partial_\theta \phi, \partial_\rho \phi, \partial_y \phi, \partial_z \phi), & (31)
\end{aligned}
$$

Above, $\phi := \phi^{(nk\lambda)}(x)$ previously defined, $k := |\mathbf{k}|$ and ϵ_{ij} is the Levi-Civita pseudo-tensor on $I\!\!R^2$ in Cartesian coordinates. The first three eigenfunctions satisfy $\nabla^a A_a = 0$ and have eigenvalue $\varepsilon_I^{(\eta)} = \varepsilon_{II}^{(\eta)} = \varepsilon_{III}^{(\eta)} = -(\lambda^2 + \mathbf{k}^2)$, while $A_a^{(IV)}$ has eigenvalue $\varepsilon_{IV}^{(\eta)} = -(\lambda^2 + \mathbf{k}^2)/\eta$. Few calculations, involving only Eq.(19) and the form of the employed modes in terms of the scalar modes, lead quite easily us to[3]:

$$G_{\beta\, aa'}^{(\eta)}(x,x') \quad := \quad \int_0^{+\infty} d\lambda \int_{I\!\!R^2} d\mathbf{k} \sum_{n\in Z\!\!\!Z} \sum_{m=I}^{IV} \frac{A_a^{(m,n\lambda\mathbf{k})}(x) A_a^{(m,n\lambda\mathbf{k})*}(x')}{\varepsilon_y^{(\eta)}} = \tag{32}$$

$$= \quad G_{\beta aa'}(x,x') + (\eta-1)\partial_a \partial_{a'} \int d\lambda \int d\mathbf{k} \sum_{n\in Z\!\!\!Z} \frac{\phi^{(n\lambda\mathbf{k})}(x)\phi^{(n\lambda\mathbf{k})*}(x')}{(\lambda^2 + \mathbf{k}^2)^2} . \tag{33}$$

Hence, due to Eq.s (25) and (18) we have our main result:

$$G_{\beta\, aa'}^{(\eta)}(x,x') = G_{\beta aa'}(x,x') + (\eta-1)\partial_a \partial_{a'}[G_\beta * G_\beta](x,x') , \tag{34}$$

or, equivalently:

$$G_{\beta\, aa'}^{(\eta)}(x,x') = S_{\beta aa'}(x,x') + (\eta-1)\partial_a \partial_{a'}[S_\beta * S_\beta](x,x') . \tag{35}$$

Denoting by $\mathbf{s}$ the three spatial variables, the previous convolution is defined as

$$[f * f'](x,x') := \int d^4y \sqrt{g(y)}\, f(x,y) f'(y,x') = \int d^3\mathbf{s}\sqrt{g(\mathbf{s})} \int_0^\beta d\theta\, f(x,(\mathbf{s},\theta)) f'((\mathbf{s},\theta),x') ,$$

Despite of a different manifold structure and a different choice for the Euclidean time, we have found the same structure of η-parametrized Green functions holding in the Euclidean section of Minkowski's space for the vacuum η-parametrized Minkowskian Green functions.

4   Discussion on a Remaining Gauge Ambiguity

Few remarks on Eq.(34). Let us prove that, in the case $\eta \neq 1$, the same features of the case $\eta = 1$ [11, 2] re-appear. $G_{\beta\, aa'}(x,x')(= S_{\beta\, aa'}(x,x'))$ which appears in Eq.(34) contains the two static terms $1/4\pi\beta'$ and $1/4\pi\rho\rho'\beta$ evaluated at $\beta' = \beta$, respectively in $S_{\beta\, \theta\theta'}(x,x')$ and $S_{\beta\, \rho\rho'}(x,x')$. In our Euclidean approach these *static* terms arise from the zero modes $\varphi^{(0\lambda)}(\rho)$ of the Laplacian on the pure cone. In a different context, Kay and Studer pointed out other subtleties related to these zero modes [14].

We can write such terms by a pure gauge term form as

$$\delta G_{\beta'\, aa'}(= \delta S_{\beta'\, aa'}) := \partial_a \partial_{a'} \Phi(x,x')_{\beta'=\beta} ,$$

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[3]Primed index derivatives act on primed arguments.

where

$$\Phi(x, x')_{\beta'} := (4\pi\beta')^{-1}(\theta\theta' + \ln\rho\ln\rho') \,.$$

Thus, the Green function for the strength field obtained from vectorial Green function employing the definition:

$$F_{ab} := \nabla_a A_b - \nabla_b A_a = \partial_a A_b - \partial_b A_a$$

and the corresponding physical quantities as the stress tensor already calculated in [11], do not depend on the value of β' as well as on the value of η.

Furthermore, we may quite simply obtain[4]

$$\nabla_a\nabla^a\Phi(x, x')_{\beta'} = 0 \quad \text{and} \quad \nabla_b\nabla^b\delta G_{\beta' \, aa'} = 0 \,.$$

Due to these properties, non depending on the values of $\beta'(\neq \beta$ in general), $\delta G_{\beta' \, aa'}$ gives no contribution in verifying the $\eta-$Green equation:

$$[\delta_a^b\nabla^c\nabla_c - (1 - \frac{1}{\eta})\nabla_a\nabla^b]G_{\beta \, bb'}(x, x') = -g_{ab'}(x) \, g(x)^{-1/2} \, \delta(x, x') \,. \tag{36}$$

Similarly it gives no contribution in verifying the $(\eta = 1)$-Ward identity necessary for the BRST invariance[5] [11, 2]

$$\nabla^a G_{\beta \, aa'}(x, x') + \nabla_{a'}G_\beta(x, x') = 0 \,. \tag{37}$$

The above identity can be proved using directly the definitions in Eq.s (19), (25) and Kabat's modes (12)-(15) (without expliciting the particular expression of the scalar eigenfunctions). Starting from Eq.(37), non depending on the value of β', due to Eq.(9) and Eq.(34), the $\eta-$Ward identity (also necessary for the BRST invariance):

$$\nabla^a G_{\beta \, aa'}^{(\eta)}(x, x') + \eta\nabla_{a'}G_\beta(x, x') = 0 \tag{38}$$

can be proved.

Hence, as in the case $\eta = 1$, the term $\delta G_{\beta' \, aa'}$ represents a remaining static gauge ambiguity which does not affect the physics non depending on the value of β'.

Few words on the mathematical structure of the found Green functions.

As it was discussed in [2], if $\beta \neq 2\pi$ no value of β' produces an everywhere defined Green function due to the conical singularity at ρ (or $\rho') = 0$, thus no value of β' is preferred in a mathematical context. Conversely, when $\beta = 2\pi$, the choice $\beta' = \beta$, and *only* that, produces a Green function well-definable on the whole manifold.

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[4]Remind that covariant derivatives commute due to the flatness of the manifold.

[5]In this context the scalar Green function G_β has to be interpreted as the ghosts Green function.

5   Vacua Coincidence at $\beta = 2\pi$

Let us address ourselves to what happens to the previous Green functions at $\beta = 2\pi$. Remind that, as found in [2]

$$S_{2\pi\ aa'}(x, x') = S_{\infty\ aa'}^{\text{Minkowski}}(x, x') + \delta S_{2\pi\ aa'}(x, x') \,.$$

where the previous Minkowski Schwinger function is obtained by the canonical quantization and is referred to the Minkowski vacuum. Taking into account that conversely

$$S_{2\pi}(x, x') = S_{\infty}^{\text{Minkowski}}(x, x')$$

and reminding the well-known form of the Euclidean Minkowski vacuum Green function in any covariant gauge, we obtain:

$$G_{2\pi\ aa'}^{(\eta)}(x, x') = S_{\infty\ aa'}^{(\eta)\ \text{Minkowski}}(x, x') + \delta G_{2\pi\ aa'}(x, x') \,.$$

Notice that

$$\delta G_{\beta'\ aa'} = \partial_a \partial_{a'} \Phi(x, x')_{\beta'} \,,$$

as well as the convolution term in Eq.(34), gives a vanishing result due to the divergence theorem, working as an integral kernel on compact support smooth test functions $J^a(x)$ satisfying $\nabla_a J^a(x) = 0$. Thus, the broken coincidence of Green functions is restored in the sense of distributions working on vanishing divergence test functions. The Lorentzian analogue of this fact is the distributional coincidence of the Lorentzian Green functions working on three-smeared wavefunctions of Lorentz and physical photons within a Gupta-Bleuler-like formalism (see [11] and [2] and also Appendix C therein).

6   Generalizations in more General Manifolds

Generalizations to manifolds as $\mathcal{M} \times I\!\!R^2$ endowed with the natural product metric are straightforward. We can consider $\mathcal{M}$ as a generally Euclidean curved two-manifold and define coordinates $x^\mu \equiv (\theta, \rho)$ on that. We could also suppose that ∂_θ defines a Killing vector with compact orbits of period β to be interpreted as the inverse of a temperature; however such a supposition is not so strictly necessary. For this generalized case (as well as in completely general manifolds), Euclidean Maxwell equations read

$$\left[-\Delta_1 + (1 - \frac{1}{\eta}) d_0 \delta_0 \right] A = 0 \,, \tag{39}$$

where $\Delta_1 = d_0\delta_0 + \delta_1 d_1$ is the Hodge Laplacian for 1-forms ($\delta_n := d_n^\dagger$ w.r.t. the Hodge scalar product.). The eigenfunctions of the operator appearing in the above equation can be still written as in the former right hand side of Eq.s(28)-(31), provided

$$\phi = \frac{1}{2\pi} e^{i\mathbf{kx}} \mathbf{J}_{n,\lambda}(x^\mu) \,,$$

where $\mathbf{J}_{n,\lambda}(x^\mu)$ is an eigenfunction of the 0-forms Hodge Laplacian $\Delta_0^{\mathcal{M}}$ on $\mathcal{M}$, with eigenvalue λ^2. Employing a bit of n-forms algebra, one can obtain in our manifold the same eigenvalues found in $\mathcal{C}_\beta \times \mathrm{R}^2$. Furthermore, once again $\delta_0 A^{(m)} = 0$, namely $\nabla^a A_a^{(m)} = 0$, in the cases $m = I, II, III$.

Starting from the definitions in Eq.(25) and Eq.(32) one can simply prove Eq.s (33) and (34) using the considered modes without expliciting the particular form of the scalar modes $\mathbf{J}_{n,\lambda}(x^\mu)$. Similarly, one can prove the Ward identities in Eq.(38) directly by Eq.(32). Notice that particularly trivial cases are $\mathcal{M} = \mathrm{I\!R}^2$ and $\mathcal{M} = S^1 \times \mathrm{I\!R}$. In these cases we trivially find respectively the expressions of the vacuum Minkowski Schwinger function and the thermal Minkowski Schwinger function in terms of the corresponding scalar Schwinger function and the Feynman Gauge Schwinger function.

Just a remark for the case of a completely general manifold generally different from $\mathcal{M} \times \mathrm{I\!R}^2$. Suppose that in a general manifold $G_{aa'}^{(\eta=1)}(x,x')$ and $G^{\mathrm{scalar}}(x,x')$ are Green functions of Δ_1 and Δ_0 respectively. One can simply prove, employing the n-forms theory, that $G_{aa'}^{(\eta)}$ given in Eq.(34) (where $G_{\beta\,aa'} \to G_{aa'}^{(\eta=1)}$ and $G_\beta \to G^{\mathrm{scalar}}$) both can still be a Green function of the operator in Eq.(39) and can still satisfy the $\eta-$Ward identity of Eq.(38). In fact, this holds if, and only if, the former two Green functions satisfy the $\eta = 1$ Ward identity of Eq.(37) which now reads

$$\delta_0 G^{(\eta=1)}(x,x') - d_1' G^{\mathrm{scalar}}(x,x') = 0 \,.$$

Acknowledgments

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